

# Prediction of the Vibration Behaviour of Structures with Joints

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## 1 INTRODUCTION

For lightly damped, linear members of a structure, very good estimates of eigenfrequencies, modal damping values, and corresponding mode shapes can be achieved by Experimental Modal Analysis (EMA). Furthermore, by model updating of finite element models of members, very good predictions of the vibration behaviour up to high frequencies are possible.

If we now assemble single members into a built-up structure, prediction of the structural vibration behaviour can be quite involved, even though the behaviour of all single members is well-known [1]. This is due to the fact that the mechanical contact at joint interfaces is usually not modelled sufficiently. Effects like uneven contact pressure distributions over the contact area, microslip damping and gapping of contact regions remain unconsidered but these effects can have a major influence on the structural vibration behaviour.

This paper introduces an improved approach to predict the vibration behaviour of built-up structures. The approach consists of two parts: (1) The development of a suitable, new contact model which is based on Greenwood's statistical approach for rough surfaces and a generalization of Mindlin's microslip model and (2) the implementation of the contact model in commercial finite element software which is done by programming a special isoparametric contact element, the so-called zero thickness element.

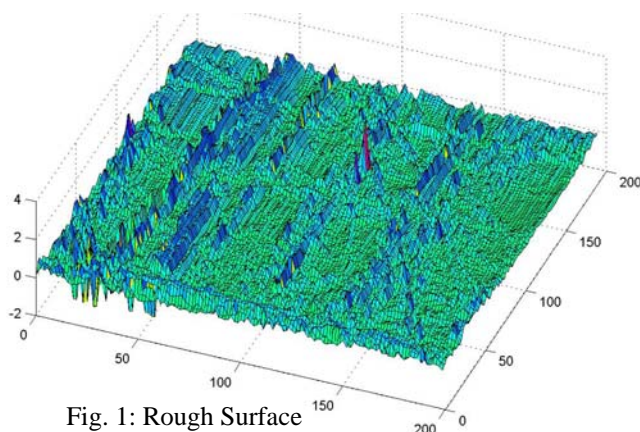


Fig. 1: Rough Surface

## 2 CONTACT MECHANICS

### 2.1 Normal Contact

Greenwood and Williamson [2] developed a model to describe the contact of a rough elastic with a planar rigid surface. The height distribution and the Abbott-curve is approximated here by an exponential distribution function. This type of distribution is usually sufficient to describe the uppermost 25 % of a measured Abbott-curve [2] and leads to a relatively simple mathematical relation between normal pressure and relative displacement:

$$p_N = p_{N0} \exp(-(g_N - g_{N0}) / \sigma).$$

The models for normal and tangential contact are explained in more detail in [3].

### 2.2 Tangential Contact

The introduced model is a new hysteresis model defined by an evolution equation which is based on Mindlin's approach for tangential contact of two spheres. This evolution equation is valid for increasing tangential loading. For decreasing loading the relation between tangential force and tangential relative displacement is assumed to follow a linear elastic law,

Loading:  $\dot{F}_T = k_{T0} \dot{g}_T (1 - |F_T|/(\mu F_N))^n$ ;      Unloading:  $\dot{F}_T = k_{T0} \dot{g}_T$ .

### 3 ZERO THICKNESS CONTACT ELEMENT

The element consists of two four node quadrilateral elements which face each other. In the element, the three-dimensional relative displacement field  $\{g\}$  is approximated by bilinear shape functions [H].

$$\{g\} = [H] \left( \{u\}_{nodal}^{top} - \{u\}_{nodal}^{bottom} \right).$$

The traction vector  $\{t\}$  in each element describes the interface stresses and we know from Newton's third law that the tractions must be equal in magnitude and opposite in direction.

With this we can state the virtual work of the contact tractions,

$$\delta W_C = \iint_S \delta \{g\}^T \{t\} dS.$$

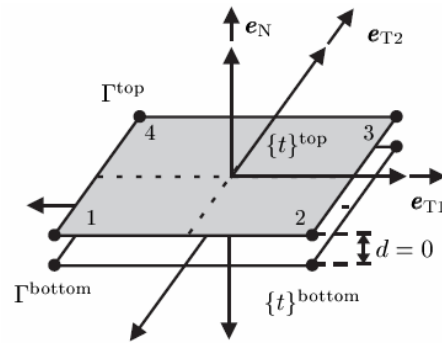


Fig. 2: Contact Element

### 4 APPLICATION

The prediction quality is shown by comparison of resonance frequencies and modal damping values of a simplified control unit up to 2 kHz determined by measuring the impulse response and following Experimental Modal Analysis and by simulation of the impulse response, Fourier transform and following Modal Analysis (see Table). The contact areas of the structure are modelled with the zero thickness elements and the introduced contact model.

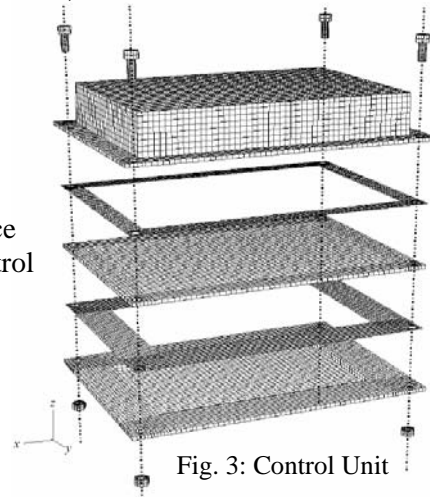


Fig. 3: Control Unit

Mode No.	1	2	3	4	5	6	7	8
Experiment	889 Hz	1101 Hz	1349 Hz	1424 Hz	1521 Hz	1645 Hz	1766 Hz	1960 Hz
	D=1.2 %	D=0.8 %	D=1.1 %	D=0.9 %	D=0.7 %	D=0.6 %	D=0.7 %	D=0.6 %
Simulation	877 Hz	1113 Hz	1366 Hz	1386 Hz	1537 Hz	1660 Hz	1753 Hz	1982 Hz
	D=0.9 %	D=0.6 %	D=0.7 %	D=0.7 %	D=0.5 %	D=0.5 %	D=0.6 %	D=0.4 %

### REFERENCES

- [1] Gaul, L. & Lenz, J., Nonlinear dynamics of structures assembled by bolted joints. *Acta Mechanica*, **125**, pp. 169–181, 1997.
- [2] Greenwood, J.A. & Williamson, J.B.P., Contact of nominally flat surfaces. *Proc. of the Royal Society of London, Series A* **295**, pp. 300–319, 1966.
- [3] Gaul, L. & Mayer, M., Efficient modelling of contact interfaces of joints in built-up structures. *Surface Effects and Contact Mechanics VIII*, WIT Press, pp. 195 – 205, 2007.

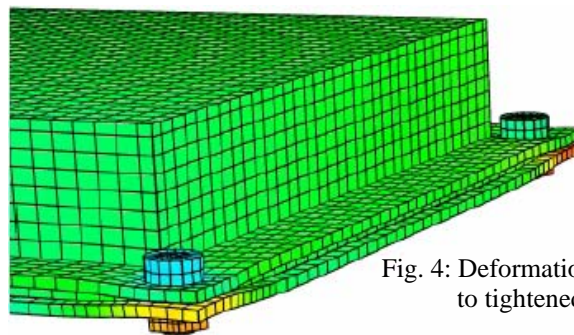


Fig. 4: Deformation due to tightened bolts