Symposium of Advances in Contact Mechanics: a tribute to Prof. J.J. Kalker
22 - 24 October 2008, Delft, The Netherlands

The role of contact mechanical model on the overall behaviour of granular materials

F. Nicot
Cemagref, Grenoble, France

F. Darve
INPG-L3SR, Grenoble, France

Keywords: granular materials, flow rule, bifurcation, second-order work, micromechanics

1 INTRODUCTION

Granular materials can be indifferently described as a set of granules, or a set of idealized punctual contacts. In most micromechanical approaches used to model the behavior of granular materials, the latter description is adopted. Assuming that granules are sufficiently rigid, this choice is motivated by the fact that the deformational processes taking place within a granular assembly are mainly directed by the relative motion between grains in contact: sliding, rolling, opening and closure are the main kinematical local ingredients responsible for the overall mechanical response of the specimen.

In the following, particular attention will be paid to the notion of flow rule and bifurcation, and their close relation with the sliding condition acting on contacts.

2 EXISTENCE OF A REGULAR FLOW RULE

In the context of the classical elastoplasticity theory, when plastic strains develop, the direction of plastic strains is assumed to be independent of the loading stress direction, and is given by a regular flow rule:

$$d\boldsymbol{\varepsilon}^p = d\lambda \frac{\partial g}{\partial \boldsymbol{\sigma}} \tag{1}$$

where $g\left(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}^p\right) = 0$ is the equation of a closed smooth surface in the stress space (called a plastic potential), and $d\lambda$ is a scalar plastic multiplier that can be determined by invoking the so-called consistency condition ($df = 0$, where f is the yield function). Eq. (1) implies that the plastic flow is perpendicular to the plastic potential *g*, irrespective of the loading directions of $d\boldsymbol{\sigma}$. In other words, once the mechanical state of a material point belongs to the yield surface, the direction of the plastic strains is independent of the loading direction. The notion of a regular flow rule as seen in the above is thus a basic ingredient of standard elastoplasticity theory. However, this feature is far from being general, and examples of materials that do not verify a regular flow rule along certain loading paths can be found.

The question of flow rule can also be raised through the dependence of the plastic strain ratios $\lambda_2^p = d\varepsilon_2^p / d\varepsilon_1^p$ and $\lambda_3^p = d\varepsilon_3^p / d\varepsilon_1^p$ over the stress loading ratios $\mu_2 = d\sigma_2 / d\sigma_1$ and $\mu_3 = d\sigma_3 / d\sigma_1$. It can be shown (Nicot and Darve, IJP, 2007) that in the general three-dimensional conditions, parameters λ_i^p depend on μ_j. This feature is related to the fact that the direction of sliding, for each contact, is not known a priori and is given by the direction of the applied tangential force.. On the other hand, in the particular two-dimensional conditions (including also the axisymmetric case), the direction of sliding for each contact is entirely prescribed: a regular flow rule exists on each sliding contact. As a consequence, in this particular case, parameters λ_i^p are no longer dependent on μ_j : the flow rule is regular.

3 THE OCCURRENCE OF BIFURCATION

3.1 *Theoretical framework*

The question on the occurrence of failure is of paramount importance in engineering. For non associated materials such as geomaterials, it is well known that various failure modes may occur before the standard plastic limit criterion is met. In a recent approach, the notion of failure is related to the increase in kinetic energy under constant loading conditions (loss of sustainability, Nicot and Darve, IJSS, 2007). It is shown that such a mechanism can be detected by the vanishing of the second-order work, defined as the inner product between incremental strain and stress tensors: neglecting geometrical changes, $W_2 = \delta\sigma_{ij}\ \delta\varepsilon_{ij}$, where $\delta\sigma_{ij}$ and $\delta\sigma_{ij}$ are related through the constitutive relation. Let us assume that a certain incremental stress loading (direction) $\delta\sigma_{ij}$ exists, such that $W_2 < 0$. Then, loading parameters can be built such that, once maintained constant, the application of an infinitesimal perturbation leads to the brutal collapse of the material. This theoretical framework was confirmed by laboratory tests and numerical simulations (using a discrete element method) recently carried out (Sibille et al., IJNAMG, 2007; Darve et al., CRAS, 2007).

3.2 *Microstructural approach*

As far as each contact between adjoining particles can be regarded as the fundamental constitutive unit of a grain assembly, a basic correspondence can be derived between the macroscopic second-order work and the discrete second-order work. The former quantity is defined from tensorial variables, whereas the latter is defined as the summation over all the contacts of the microscopic second-order works expressed from discrete variables on the scale of each contact. Considering that the contact between two granules can be described by an elastic-plastic model including a Coulomb criterion, a micro-mechanical investigation of the origin of the vanishing of the second-order work was undertaken by making use of this relation. Two origins can be distinguished. A first origin is material, and is related to the elasto-plastic behavior of contacts. In particular, the vanishing of the microscopic second-order work at a given contact requires, in two-dimensional conditions, that this contact behave in a plastic regime under normal unloading. This is no longer true in more general three-dimensional conditions, since the microscopic second-order work may remain positive for contacts in a plastic regime with normal unloading. An additional condition on the amplitude of the tangential incremental displacement must be adjoined. A second origin is purely geometrical, and is related to the sudden opening of existing contacts. Such phenomena are likely to occur within the strong phase of the medium, along the so-called force chains, inducing significant structural rearrangements.

Finally, it is worth noting the analogy between the incremental stress directions along which the second-order work takes negative values (both principal components of $\delta\sigma$ are negative), and the local conditions (both normal and tangential components of the incremental contact force δF are negative) (Fig. 1).

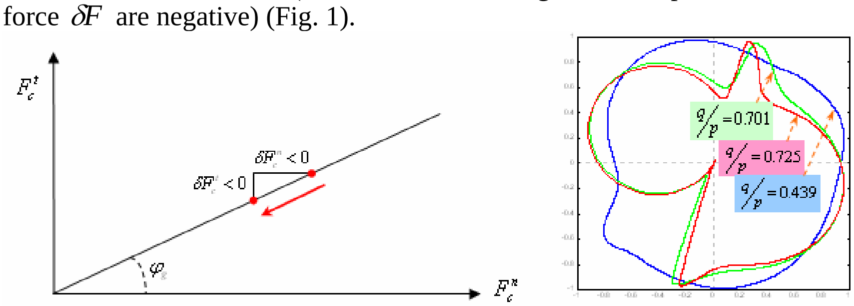

Fig. 1. (a) Evolution of the contact force for the vanishing of the microscopic second-order work: the contact force descends the Coulomb line. (b) Polar representation of the second-order work along incremental stress direction, using the micro-directional model for different deviatoric ratios.