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New mortar/finite element algorithms for large sliding contact analysis

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1 OVERVIEW

Numerical approaches to contact and impact problems have been extensively studied in recent decades, as necessitated by the great number of practical applications exhibiting these phenomena, and the need in many cases to extend beyond the capabilities provided by analytical or semi-analytical approaches. References [1]-[4] provide fairly comprehensive coverage of the important lines of research in this area, mostly within a finite element context.

Although the progress described by these and related works is impressive, a key limitation of many widely used analysis packages is the robustness with which such applications can be solved in practice. Although the reasons for this are several, probably the most important is that the constraints and physics imposed by contact interactions are strongly nonlinear and more importantly, nonsmooth. Canonical numerical approaches based on continuum mechanical formulations tend to assume some degree of temporal and spatial smoothness in the solution, and in fact such smoothness is relied upon quite often for effectiveness of solvers and iteration methods used to recover solutions.

Accordingly, a number of recent works (see, for example, refs [5]-[7]) have explored the extension of so-called mortar methods, originally proposed for domain decomposition, as an alternative spatial formulation of contact constraints. This construction is particularly advantageous when used with the variationally-based finite element method, and can be shown (at least when solutions possess a certain degree of regularity) to preserve the convergence rates associated with the underlying numerical method. Particular in multi-body contact problems, where deformability and complex response is expected on both sides of the contact interface, the variational consistency associated with these approaches can produce considerably more accurate and reliable results than more classical, ad-hoc nodal constraint strategies.

These results were anticipated from the prior application of these methods to domain decomposition, but our recent development of these methods (see refs. [8]-[12]) has revealed another highly desirable attribute; namely, that their nonlocal nature produces much more robust numerical formulations from a nonlinear equation solving context. The effect of this is that quite often, problems are solvable using mortar-based contact methodology that do not even produce convergent results in a more classical formulation—particularly when both deformations and relative sliding distances are large. This talk will explain the fundamental concepts underlying this technique of numerical approximation, and will provide several examples of this robustness through a series of representative simulations.

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